

Rossmoyne Senior High School

Semester One Examination, 2018

Question/Answer booklet

MATHEMATICS APPLICATIONS UNIT 3 AND 4

Section Two:

Calculator-assumed

If required by your examination administrator, p	lease
place your student identification label in this I	XOC

Student number:	In figures	
	In words	
	Your name	

Time allowed for this section

Reading time before commencing work: ten minutes

Working time: one hundred minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet Formula sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,

correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper,

and up to three calculators approved for use in this examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	10	10	100	98	65
				Total	100

Instructions to candidates

- 1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.
- 2. Write your answers in this Question/Answer booklet.
- 3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.
- 4. Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
- 5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
- 6. It is recommended that you do not use pencil, except in diagrams.
- 7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section Two: Calculator-assumed	65% (98 Marks)

This section has **ten (10)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 9 (7 marks)

- (a) The first three terms, in order, of a geometric sequence are 1400, 980 and 686.
 - (i) Deduce a rule for the n^{th} term of this sequence.

(2 marks)

(ii) Calculate the 4^{th} term of the sequence.

(1 mark)

- (b) The first three terms, in order, of an arithmetic sequence are 2.4, 5.5 and 8.6.
 - (i) The rule for the n^{th} term of this sequence can be written in the form $T_n = pn + q$. Determine the values of p and q. (3 marks)

(ii) Calculate the 177^{th} term of the sequence.

(1 mark)

Question 10 (10 marks)

A public relations company was tasked with determining whether a person's support for a sugary drinks tax could be associated with their interest in the news.

The company carried out a telephone survey, where people could respond to two questions as shown in the following table:

Question	Choice of response
Are you interested in the news?	Yes or No
Do you support a sugary drinks tax?	Yes, No or Undecided

The responses to the telephone survey are summarised in this table:

		Support for a sugary drinks tax			
		Yes	No	Undecided	
Interest in news	Yes	528	374	221	
	No	196	204	146	

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(i) answered no to being interested in the news. (1 mark)

(ii) responded to the survey. (1 mark)

(b) If there was an association between interest in the news and support for a sugary drinks tax, should the company expect the same percentage of those who support a sugary drinks tax to be interested in the news compared to those who do not support a sugary drinks tax? Explain your answer. (2 marks)

(c) Complete the two-way table below to show the associated **row** percentages for the previous table, rounding percentages to the nearest whole number. (3 marks)

		Support for a sugary drinks tax			
		Yes	No	Undecided	
Interest in	Yes				
news	No				

(d) What percentage of those who are interested in the news support a sugary drinks tax?

(1 mark)

(e) In the context of the task they were given, how should the public relations company interpret the responses to their survey? (2 marks)

Question 11 (9 marks)

To save money towards a deposit on a house, Anh and Bo started an investment account. They made an initial deposit of \$3 700, and then deposited an extra \$850 at the end of each month for the next year. Interest on the account was to be calculated and paid monthly.

The table below shows the progress of their savings for the first few months.

Month	Balance of	Interest added	Deposit made	Balance of
	account at start	at end of month	at end of month	account at end
(n)	of month (T_n)			of month (T_{n+1})
1	\$3 700.00	\$14.80	\$850.00	\$4 564.80
2	\$4 564.80	\$18.26	\$850.00	\$5 433.06
3	\$5 433.06	\$21.73	\$850.00	\$6 304.79
4	\$6 304.79	Α	\$850.00	В

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(a)	What was the	e monthly percenta	ge interest rate?			(1 mark)
(b)	Determine the	e values of A and E	3 in the table.			(2 marks)
(c)	Write a recur	sive rule to determi	ine the balance of t	he account at the	start of each	n month. (2 marks)
(d)	Determine, to	the nearest dollar	, the balance of the	e account at the sta	art of month	13. (2 marks)

(e) Calculate, to the nearest dollar, the total interest earned up to the start of month 13. (2 marks)

Question 12 (7 marks)

An employer is considering two pay schemes for new employees:

Scheme A

For the first month, a new employee is paid \$750. In subsequent months, their pay is increased so that it is \$40 more than the previous month.

Scheme B

For the first month, a new employee is paid \$750. In subsequent months, their pay is increased so that it is 5% more than the previous month.

(a) Write a recursive rule for A_n , the payment made in month n to an employee using Scheme A and another recursive rule for B_n , the payment made in month n to an employee using Scheme B. (3 marks)

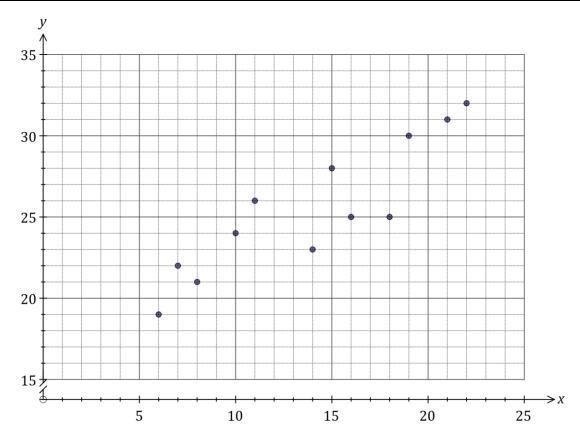
(b) In which month does the payment using Scheme B first exceed that using Scheme A, and by how much? (2 marks)

(c) If an employee only worked for 6 months, which pay scheme should they choose in order to gain the most pay? Justify your answer. (2 marks)

Question 13 (15 marks)

A tomato grower added varying amounts of a liquid fertiliser (x ml) to the irrigation systems of twelve greenhouses and observed the resulting yield of tomatoes per plant (y kg). A sample of the data recorded is shown in the table and scatterplot below.

<i>x</i> (mL)	6	7	8	10	11	14	15	16	18	19	21	22
y (kg)	19	22	21	24	26	23	28	25	25	30	31	32



- (a) Name (in words) the response variable. (1 mark)
- (b) For this data, calculate

(i) the correlation coefficient. (1 mark)

(ii) the values of a and b in the equation of the least-squares line y = ax + b clearly stating which is which. (2 marks)

(c)	What percentage of the variation in the yield per plant can be explained by	the variation in
	the amount of liquid fertiliser added?	(1 mark)

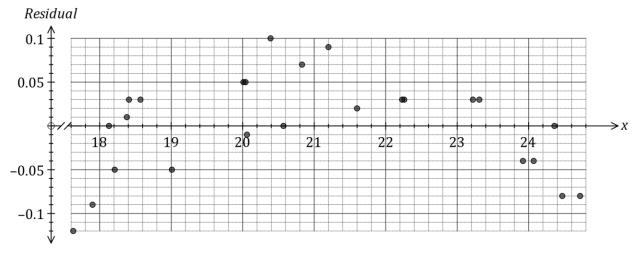
(d) If the amount of liquid fertiliser added to the irrigation system in a greenhouse was increased by one millilitre, what increase in the yield of tomatoes per plant can be expected? Explain your answer. (2 marks)

- (e) If no liquid fertiliser was used, what yield of tomatoes per plant does the linear model predict? (1 mark)
- (f) Use the equation of the least-squares line to calculate the value of y when x = 4 and when x = 24. (2 marks)

- (g) Use your answers to part (f) to draw the least-squares line on the scatterplot. (2 marks)
- (h) Estimate the yield of tomatoes per plant when 13 ml of liquid fertiliser is added to the irrigation system and comment on the reliability of this estimate. (3 marks)

Question 14 (10 marks) A painting was sold for \$30 000 by an art gallery. The painting could be bought on terms in which a premium of 7.5% was added to the purchase price and then the total amount repaid to the gallery in 24 equal monthly repayments. How much would each repayment be? (2 marks) (b) The painting was expected to increase in value at an annual rate of 9%. What is the expected value of the painting 4 years after it was purchased, to the nearest one hundred dollars? (2 marks) An amount of \$30000 can so be invested using two different saving schemes: (c) Scheme A, in which interest of 6.21% per annum is compounded monthly; and Scheme B, in which interest of 6.29% per annum is compounded semi-annually. (i) Calculate the effective interest rates of both schemes, correct to 4 decimal places, and hence state which scheme would pay the most interest over three years. (3 marks) (ii) Calculate the interest that accrues over three years on the \$30000 using the scheme which pays the most interest. (3 marks) Question 15 (8 marks)

A linear model was fitted to a set of data, resulting in a correlation coefficient of r=0.94 and a least-squares line with equation $\hat{y}=5.16+0.16x$. A residual plot for the linear model is shown below.



(a) Calculate, and add to the plot above, the residual for the point x = 22, y = 8.76. (3 marks)

(b) Use the residual plot to comment on the appropriateness of fitting a linear model to the data. (2 marks)

(c) Determine the y-coordinate of the point with a residual of 0.07 on the above plot. (3 marks)

Question 16 (13 marks)

When Atarcoin, a new cryptocurrency was launched, one Atarcoin was valued at \$4.00. After one week of trading, the value of Atarcoin had increased to \$5.00, and after another week had increased to \$6.25.

(a) Show that the value of Atarcoin increased by 25% each week.

(2 marks)

The value of Atarcoin, V_n in dollars, n weeks after its launch date, can be modelled by the recurrence relation $V_{n+1} = 1.25V_n$, $V_0 = 4$.

(b) Calculate the value of Atarcoin ten weeks after its launch date.

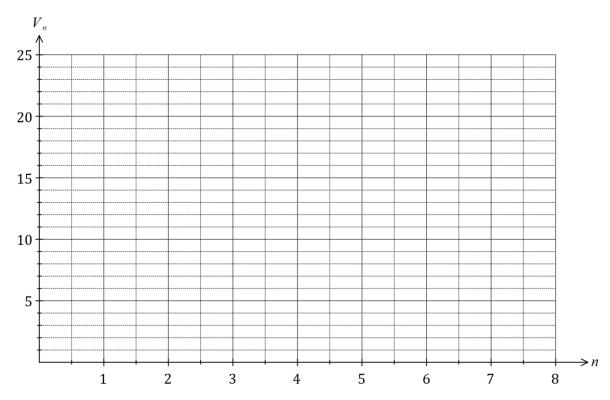
(1 mark)

(c) At the end of which week did the value of Atarcoin first exceed \$100?

(1 mark)

(d) Graph V_n against n on the axes below.

(3 marks)



See next page

The value of Atarcoin peaked at the end of week 38, and from that time onwards, its value fell by 30% each week.

(e) Determine the value of Atarcoin at the end of week 39.

(2 marks)

(f) Ignoring any fees involved in buying and selling a cryptocurrency, determine the profit or loss made by a person who bought 25 Atarcoins eight weeks after their launch, held them for 40 weeks and then sold them all. (4 marks)

Question 17 (9 marks)

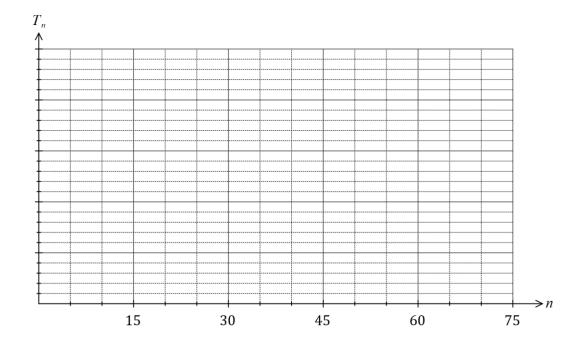
The temperature, ${}^{\circ}C$, of an industrial oven n minutes after it is turned on can be modelled by

$$T_{n+1} = 0.92T_n + 19.2, T_0 = 15$$

(a) Use the recurrence relation to complete the table of values below, rounding the temperature to the nearest °C. (2 marks)

n	0	10	20	30	40
T_n					

(b) Sketch a graph of the temperature of the oven for the first 75 minutes on the axes below. Make sure you add a suitable scale to the vertical axis. (4 marks)



(c) The manufacturer claims that the oven will reach within $10\,^{\circ}\text{C}$ of its maximum temperature within $30\,^{\circ}\text{minutes}$ of being turned on. Comment on this claim. (3 marks)

Question 18 (10 marks)

An annuity paying a monthly sum of \$2 500 is set up with an initial sum of \$500 000 and interest of 6.2.% per annum compounded monthly.

(a) The balance of the loan at the start of month n is given by the recurrence relation $A_{n+1} = rA_n - d$, $A_1 = 500000$. State the values of r and d. (2 marks)

(b) Determine the value of the annuity after twelve months and comment on what this figure indicates. (2 marks)

- (c) Determine to the nearest dollar, the monthly sum that should be withdrawn from the annuity if
 - (i) The annuity is to last for 20 years. (1 mark)

(ii) The annuity is to be a perpetuity.

(d) If the interest rate of 6.2% was halved after one year, calculate the total interest accrued by the annuity over the first two years. (3 marks)

(1 mark)

Supplementary page